UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

(2)

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

DTIC FILE COPY

AD-A204 406

1a. REPORT SECURITY CLASSIFICATION	
1b. RESTRICTIVE MARKINGS	
3. DISTRIBUTION/AVAILABILITY OF REPORT	Approved for public release; distribution unlimited.
5. MONITORING ORGANIZATION REPORT NUMBER(S)	AFOSR-TR- 89-0014
6a. NAME OF PERFORMING ORGANIZATION	Rensselaer Polytechnic Inst. MathematicalSciences Dept.
6b. OFFICE SYMBOL (If applicable)	
7a. NAME OF MONITORING ORGANIZATION	AFOSR
6c. ADDRESS (City, State, and ZIP Code)	Troy, New York 12180
7b. ADDRESS (City, State, and ZIP Code)	Building 410 Bolling, AFB DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION	AFOSR
8b. OFFICE SYMBOL (If applicable)	NM
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	AFOSR 86-0180
8c. ADDRESS (City, State, and ZIP Code)	Building 410 Bolling, AFB DC 20332-6448

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61102F	2304	A1	

11. TITLE (Include Security Classification)

NONLINEAR ANALYSIS IN INVERSE PROBLEMS AND CONTROL

12. PERSONAL AUTHOR(S)

Joyce R. McLaughlin

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
FINAL	FROM 31Jul86 TO 31Aug88	5 July 88	11

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A major activity is in inverse nodal problems, a new class of inverse problems. This work is focused on uniqueness results, algorithms and bounds for the algorithms and bounds for the algoithms. These new results show strong uniqueness thorems and (surprisingly) accurate algorithms with a mimium amount of data. Another major area of research a has been obtained for inberse spectral theory problems. Existence and uniqueness results are sought, in current projects,, for inverse membrane problems and for one dimensional problems with 'rough' coefficients. Accurate approximations are sought of continuous invers problems by discretized inverse problems. (KR)

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	Unclass

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL
Major James Crowley	(202) 767- 5027	NM

DD Form 1473, JUN 86 Previous editions are obsolete. SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

AFOSR-TR- 89-0014

Final Report

"Nonlinear Analysis in Inverse Problems and Control"

AFOSR Grant No. AFOSR-86-0180

Accession For	
NTIS GRA&I	X
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

QUALITY INSPECTED 4

Principal Investigator: Joyce R. McLaughlin
Mathematical Sciences Department
Rensselaer Polytechnic Institute
Troy, New York 12180

89 2 10 111

Introduction

Professor Joyce McLaughlin's research projects are in inverse problems and optimization. A major activity is in inverse nodal problems, a new class of inverse problems. This work is focused on uniqueness results, algorithms and bounds for the algorithms. These new results show strong uniqueness theorems and (surprisingly) accurate algorithms with a minimum amount of data. Another major area of research has been inverse spectral theory. Here, stability results have been obtained for inverse spectral theory problems. Existence and uniqueness results are sought, in current projects, for inverse membrane problems and for one dimensional problems with 'rough' coefficients. Accurate approximations are sought of continuous inverse problems by discretized inverse problems.

A new area for Professor McLaughlin, joint with her Ph.D. student, Steve Cox, is homogenization and optimization. Unique, composite membranes which maximize a given eigenvalue have been characterized.

Student Steve Cox has finished his thesis work this summer. Professor McLaughlin has four additional Ph.D. students.

All of the completed and ongoing projects are described in this report.

Inverse Nodal Problems

Inverse nodal problems were defined and the first uniqueness result was published by Professor McLaughlin in [4]. Now Professor McLaughlin, in joint work with Professor Ole H. Hald, University of California, Berkeley, has produced a complete solution to a large class of second order inverse nodal problems in one dimension. The results are contained in the two attached manuscripts, [5], and [6].

Inverse nodal problems are a new class of inverse spectral problems defined as follows. Suppose we are given a vibrating system which has a discrete set of eigenvalues (or natural frequencies). If the system is excited at a natural frequency, the positions where no excitation occurs are called nodal positions. These positions can be measured. For example, in a mechanical system a lazer beam can be directed at the vibrating system. The Doppler shift of the lazer beam, due to the motion of the vibrating system can be measured. The positions where the Doppler shift is minimized are the nodal positions. The data for the inverse nodal problems are the nodal positions. The solutions of the inverse nodal problems are differential operators. More specifically the solutions are coefficients in the operators which define physical quantities such as density, stiffness, and cross sectional area.

Vibrating mechanical systems,...beams, membranes, and plates...are major applications for this work. Geophysical applications are being pursued. Biological applications also are possible. For example, nodal line patterns for human skulls have been plotted, see [8].

Parameter identification with spectral data is an efficient use of data. Historically the spectral data has been natural frequencies, boundary measurements of the mode shapes, and energy measurements of the mode shapes.

Exact solution methods, and error bounds, have been obtained for these problems, see [1], [2], and [3].

For inverse nodal problems uniqueness results for up to two coefficients have been obtained where the data is only a dense set of nodal positions. It has been shown, see [4], that a dense set of positions is obtained by making one measurement for each mode shape. The importance of these uniqueness results is to exhibit that a great deal of information is obtained from a minimum number of measurements. The proofs employ algebraic geometric ideas as in Pöschel and Trubowitz [2].

Algorithms for calculating the coefficients are presented, see [5], [6]. Here the data is a single natural frequency and all the nodal positions for the corresponding mode shape. The main results are as follows: (1) Adjacent nodal positions provide local information; (2) Algorithms use as data the length of the interval between nodal positions and provide values for the unknown coefficients in the interval between the nodal positions; (3) Algorithms for values of the unknown parameters near the boundary are different from the algorithms for values of the coefficients at interior intervals; (4) Technically intensive proofs have yielded error bounds for the numerical methods. Two methods can be used to find the bounds. In one method the vector fields are calculated and integrated. The underlying idea is to calculate the flow of the nodal positions as the unknown parameters change. The second method is based on a root finding scheme and produces an interval which contains a zero of a specific function.

Numerical calculations have been completed to show a quadratically converging method and the (amazingly) high accuracy achieved with a minimum number of measurements. A main advantage of using this data is that it is <u>not</u> computationally intensive.

There are several ongoing projects. One is to extend these results to Euler-Bernoulli and Timoshenko models for the beam. The first project is nearly completed and will be reported at the IEEE CDC meeting in Austin, Texas, in December, 1988. Detailed asymptotics for the spectral data for the Euler-Bernoulli Beam were required. These new results are also being applied by Professor McLaughlin to improve her previous results on the inverse spectral problem for fourth order operators.

Professor McLaughlin and Professor Hald now have a proposed algorithm for two dimensional partial differential equations models for a membrane and a long term project is to investigate methods for determining bounds. The bounds will be used to predict (and explain) the accuracy of the algorithm.

Homogenization Problem

Professor McLaughlin with Ph.D. student, Steve Cox, investigated the following homogenization problem. Suppose we are given a composite membrane with fixed boundary. Suppose the membrane is composed of two materials in fixed ratio, one of constant density α and the other material of constant density β , $\beta > \alpha > 0$. Determine how to distribute the two materials so that a given eigenvalue is maximized or minimized.

To put this problem in perspective we review the work of Krein [12], in one-dimension for the problem, maximize λ_m . Krein showed that the density distribution is unique. It is obtained by subdividing the interval into m subintervals of equal length. On each subinterval $\rho = \alpha$, the lighter material, on a symmetric interval centered at the midpoint; $\rho = \beta$ on the remainder of the interval. The eigenfunction, u , has nodes at the endpoints of the subintervals and is symmetric on each subinterval. The proof invokes symmetry arguements.

Now an equivalent formulation is that there exists $\ell > 0$ such that when

$|u| > \ell$, $\rho = \alpha$ and when $|u| < \ell$, $\rho = \beta$. It is this idea that is generalized for the optimal design problem for arbitrary bounded domains in R^n in the thesis of Steve Cox. He shows ρ is unique and gives the correct characterization for ρ for the optimization problem max λ_1 . The method is as follows. Using a functional $A(u , \rho)$ of Auchmuty, [10] the optimal design problem is formulated as sup inf $A(u , \rho)$. He shows that the optimal value is achieved at a saddle point. The uniqueness and characterization follow from this fundamental result. The proof applies the ideas of Knaster, Kuratowski, Mazurkiewicz, and Ekeland and Temam, see [8], [10], [12], [13].

Similar results are shown for the optimization problem min λ_m when it is known that the corresponding eigenfunction has m nodal domains. It is shown that even for the weak formulation of the eigenvalue problem in n dimensions λ_m is the first eigenvalue for each nodal domain of the corresponding eigenfunction. This last result is also important for the inverse nodal problem in two dimensions.

Steve Cox has his thesis defense on August 23, 1988. He will be a Post-Doctoral Fellow at Courant Institute for the academic year 1988-89. Beginning in the fall, 1989, Steve will be an Assistant Professor (tenure-track) at Rice University.

Stability Results for Inverse Eigenvalue Problems

Professor McLaughlin has displayed exact solution methods for second and fourth order ordinary differential operators. In this work [7] she establishes stability results for the Gel'fand-Levitan exact solution method for two second order operators. The proofs required establishing the tangent vectors for the coordinate 'lines' defined by the Gel'fand-Levitan data for the inverse problem. The basic idea was to apply the algebraic geometric approach of Pöschel and Trubowitz [2] to the Gel'fand Levitan theory.

the algebraic geometric approach of Pöschel and Trubowitz [2] to the Gel'fand Levitan theory.

The stability result established error bounds. It showed that a continuation method using vector fields was feasible. The continuation method was successfully implemented for a limited number of cases by Master's student, Kelly Belford. Professor McLaughlin, jointly with William Rundell, Texas A&M, continue numerical studies with continuation methods for inverse spectral problems.

Ongoing Projects with Ph.D. Students

Roger Knobel investigates an Inverse Spectral Problem in two dimensions. Very little is known about this problem save the uniqueness result of Sylvester and Uhlmann [17]. Roger has developed an algorithm using a Rayleigh-Ritz formulation. This is a generalization of the numerical method for inverse spectral problems in one dimension, see Hald [15]. R. Knobel proposes to develop a uniqueness theorem using a modification of the techniques which have been presented by Eskin, Ralston, and Trubowitz for the periodic problem in the plane. Roger is supported by a National Science Foundation graduate student grant.

Carol Coleman investigates the inverse spectral problem for ordinary differential equations with rough coefficients. She is extending the ideas put forth in Pöschel and Trubowitz [2] for the operator $\mathscr{L} = \frac{1}{p^2}\frac{d}{dx}\{p^2 \frac{d}{dx}\}$ where $p^2 \in L^2$ and $\frac{1}{p^2} \in L^2$. The difficulty here is that there is no known asymptotics for the spectral data. She has beautifully displayed tangent and normal vector for the isospectral (and iso-norming constant) sets. Her aim is a global existence-uniqueness result and careful treatment of continuation methods using these vector fields. This extension provides a much shorter and more elegant theoretical structure than the work of L. E. Anderson, [18].

Carol is supported by an Office of Naval Research grant (with funds from ONR and AFOSR).

Bruce Geist seeks numerical and theoretical results which show how solutions of matrix (or discretized) inverse spectral problems approximate solutions of continuous inverse spectral problems. Bruce is supported by the ONR grant (with funds from ONR and AFOSR).

C. J. Lee begins her Graduate study at Rensselaer this fall at Rensselaer Polytechnic Institute. She has just received her Masters degree at the University of California, Berkeley. She will be supported her first year by a Parson's Fellowship. She will begin preliminary work on her thesis this August.

References

[1] I. M. Gel'fand and B. M. Levitan, "On the determination of a differential equation from its spectrum," Isv. Akad. Nauk SSSR Ser. Mat., 15(1951), 309-360; Amer. Math. Soc. Transl., 1(105), 253-304.

[2] J. Poschel and E. Trubowitz, Inverse Spectral Theory, Academic Press, New York, 1986.

[3] Joyce R. McLaughlin, "Analytical methods for recovering coefficients in differential equations from spectral data," SIAM Rev. 28(1986), 53-72.

[4] Joyce R. McLaughlin, "Inverse Spectral Theory Using Nodal Points as Data - A Uniqueness Result," J. Diff. Eq. 73(1988), 354-362.

[5] Ole H. Hald and Joyce R. McLaughlin, "Inverse Problems Using Nodal Position Data - Uniqueness Result, Algorithms and Bounds," Proceedings of the 1986 Inverse Problems Program, Australian National University, Canberra, Australia (27 pp.)

[6] Ole H. Hald and Joyce R. McLaughlin, "Solutions to Inverse Nodal Problems," (50 pp) submitted to Inverse Problems.

[7] Joyce R. McLaughlin, "Stability theorems for two inverse spectral problems," Inverse Problems, 4(1988), 529-540.

[8] T. B. Khail and D. C. Viano, "Experimental Analysis of the Vibrational Characteristics of the Human Skull," J. Sound and Vibration, 63(1979), 351-376.

[9] I. Ekeland and R. Temam, Convex Analysis and Variational Problems, North Holland, Amsterdam, 1976.

[10] G. Auchmuty, "Dual Variational Principles for Eigenvalue Problems," Nonlinear Functional Analysis and its Applications, F. Browder, ed., AMS, (1986), 55-71.

[11] S. Kesavan, "Homogenization of Elliptic Eigenvalue Problems I," Appl. Math. Optm., 5(1979), 153-167).

[12] M. G. Krein, "On Certain Problems on the Maximum and Minimum of Characteristic Values and on the Lyapunov Zones of Stability," AMS Translations, Ser. 2, 1(1955), 163-187.

[13] V. Barbu and Th. Precupanu, Convexity and Optimization in Banach Spaces, Reidel, Boston, 1986.

[14] H. Attouch, Variational Convergence for Functions and Operators, Pitman, Boston, 1984.

[15] Ole H. Hald, "The Inverse Sturm-Liouville Problem and the Rayleigh-Ritz Method," Math. of Comp. 32(1978), 687-705.

References (continued)

[16] G. Eskin, J. Ralston, and E. Trubowitz, "On Isospectral Periodic Potentials in R^n," Comm. Pure Appl. Math. 37(1984), 647-676.

[17] J. Sylvester and G. Uhlmann, "A global uniqueness theorem for an inverse boundary value problem," Annuls of Math. 125(1987), 153-169.

[18] L. E. Anderson, "Inverse eigenvalue problems with Discontinuous Coefficients Inverse Problem 4(1988).

Invited Plenary Lectures (for J. McLaughlin)

Series of Three Lectures: John Barrett Lectures: University of Tennessee April, 1987

"Exact Solution Methods - Theory and Numerical Calculations - for Second Order Inverse Spectral Problems"

"Inverse Spectral Problems Using Nodal Position Data"

"Exact Solution Methods for Inverse Spectral Problems for Second Order Systems"

Workshop on Inverse Problems, Institute for Mathematics and Applications, University of Minnesota January, 1987

"Using Nodal Position Data in the Inverse Problem"

Workshop on Control of Systems governed by Partial Differential Equations, Montreal, Canada October, 1986

"Operator Splitting, Integral Equations, and O.D.E. Methods for the Inverse Timoshenko Beam"

Mathematics Colloquium, Australian National University, Canberra, Australia September, 1986

"Inverse Spectral Problems Using Nodal Positions as Data"

Applied Mathematics Semniar, Australian National Universtiy, Canberra, Australia September, 1986

"Operator Splitting as Applied to Inverse Spectral Problems"

Special Sessions and Minisymposiums Organized

Professor McLaughlin organized a minisymposium at the International ICIAM meeting in Paris, France in June, 1987.

Session Title: On the Nonlinear Dependence of Material Parameters on Spectral Data.

Speakers:

David Russell, University of Wisconsin, "Inverse Problem for the Elastic Beam with Transfer Function Data".

T. Suzuki, University of Tokyo, Japan, "A Global Analysis for a Nonlinear Eigenvalue Problem".

Ole H. Hald, University of California, Berkeley, "Numerical Calculations for Inverse Nodal Problems".

Joyce R. McLaughlin, Rensselaer Polytechnic Institute, "Uniqueness Results for Inverse Nodal Problems".

Professor McLaughlin organized the SIAM special session at the IEEE meeting on Control and Decision in Los Angeles, California in December 1987.

Session Title: Parameter Identification in Inverse Problems and Control

Speakers:

K. Bube, University of Washington, "Determining Distributed Parameters for Media with Attenuation".

M. Vogelius, University of Maryland, "Variational Approach to Determine Conductivity by Boundary Measurements".

P. Sacks, Iowa State University, "An Inverse Problem in Elastic Wave Propagation".

J. Baumeister, Frankfurt, West Germany, "Asymptotic Embedding Methods for Parameter Estimation".

J. McLaughlin, Rensselaer Polytechnic Institute, "Parameter Identification Using Nodal Position Data".

K. Murphy, University of North Carolina, Chapel Hill, "Time Dependent Approximation Schemes for Some Problems of Parameter Estimation in Distributed Systems".